Applications of Exponential Functions

There are many applications of exponential functions in business and economics. Below are examples where an exponential function is used to model and predict cost and revenue:

The function $C(t) = C_o\,(1+r)^t$ is used to model the rise in cost of a product that has a cost of C_o today and is predicted to have an average annual inflation rate of r for t years. If a house is purchased today for \$202,300 what is the predicted cost of the same house in 8 years if the annual inflation rate for the next eight years is predicted to be 3.5%?

$$C(t) = 202,300\,(1+.035)^8$$

$$= \$\,266,390.47$$

The revenue generated by selling x units of a product at a price $p(x)$ per unit is given by $R(x) = x \cdot p(x)$

If $p(x) = 34\,(4)^{-x/6}$

determine the revenue if 12 units of the product is sold.

$$R(x) = x\,p(x) = (12)(34)(4^{-2})$$

$$= \$\,25.50$$

If a populations growth is proportional to the number in the population, then we say that the population grows exponentially.

A model for exponential growth
$A(t) = A_o\, a^t$ where a is a number greater than 1.
- t is time.
- $A(t)$ is amount at time t.
- A_o is the initial Amount.

**An alternative form for this same function is* $A(t) = A_o\, e^{kt}$ *where k is a positive real number.*

If the decay of a substance is inversely proportional to the amount of substance then the substance will follow an exponential decay model.

A model for exponential decay
$A(t) = A_o\, a^t$ where a is a number less than 1.
- t is time.
- $A(t)$ is amount at time t.
- A_o is the initial Amount.

***An alternative form for this same function is* $A(t) = A_o\, e^{kt}$ *where k is a negative real number.*

If it takes 2 hours for a population of bacteria to quadruple in size:

1) Determine a so that $A(t) = A_o\, a^t$ describes the number of bacteria after t hours.

$A(0) = A_0$, $A(2) = 2A_0 = A_0 a^2$

$a^2 = 2$, $a = \sqrt{2}$

2) If a population is measured at 500,000 bacteria, how many bacteria will exist 12 hours later?

$A(t) = A_0 a^t = 500{,}000\,(\sqrt{2})^t$

$A(12) = 500{,}000\,(\sqrt{2})^{12}$

$= 32{,}000{,}000$

If the half-life of a radioactive element is 1020 years:

1) Determine a so that $A(t) = A_o\, a^t$ describes the amount of the element that remains after t years.

$A(0) = A_0$, $A(t) = A_0 a^t$

$A(1020) = \frac{A_0}{2} = A_0 a^{1020}$

$a^{1020} = \frac{1}{2}$, $a = \left(\frac{1}{2}\right)^{\frac{1}{1020}} = .99932$

2) How much of a 10 gram sample of the radio active element will remain after 5,000 years?

$A(t) = 10\,(.99932)^t$, $A(5000) = .33335$ grams

Compound Interest Formula:

An investment of P dollars at an annual interest rate of r, compounded n times per year, has an accumulated value after t years of:

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

If the interest is compounded continuously then the accumulated value is give by:

$$A(t) = P\,e^{rt}$$

When Seth was born, his parents invested \$10,000 into an account that has averaged 8% annual interest. Seth will be 65 and ready to retire on his birthday tomorrow. He has never touched any money in that investment. How much will that investment be worth on his birthday tomorrow?

$$A(t) = 10{,}000\left(1 + \frac{.08}{12}\right)^{12t}$$

$$A(65) = \$\,1{,}781{,}709.21$$

Leslie wants to earn \$1500 in interest in 5.2 years from an investment of \$11000 in an account that is compounded quarterly. What interest rate will she need in order to achieve her goal?

$$A(t) = 11{,}000\left(1 + \frac{r}{4}\right)^{4t}$$

$$11000\left(1+\frac{r}{4}\right)^{4(5.2)} = 11000 + 1500 = 12500$$

$$\left(1+\frac{r}{4}\right)^{20.8} = \frac{12{,}500}{11{,}000}$$

$$r = .0247$$

about $2\frac{1}{2}$ %